1.3　一端固定他端単純支持梁

1.3.1　集中荷重

① 反力 R

$$R_O = \frac{PL_2^2}{2L^3}(2L + L_1)$$

$$R_A = P - R_0$$

② せん断力　S

$$S_{OB} = R_O$$

$$S_{BA} = R_O - P$$

③ 曲げモーメント　M

$$M_{OB} = R_0 x$$

$$M_{BA} = R_0 x - P(x - L_1)$$

$$M_A = -\frac{PL_1L_2}{2L^2}(L + L_1)$$

$$M_{\max} = \frac{PL_1L_2^2}{2L^3}(2L + L_1) \qquad (x = L_1)$$

$$M_{\min} = -\frac{PL_1L_2}{2L^2}(L + L_1) \qquad (x = L)$$

④ たわみ　δ

$$\delta_{OB} = \frac{1}{6EI}\left\{R_0(3L^2x - x^3) - 3PL_2^2x\right\}$$

$$\delta_{BA} = -\frac{1}{6EI}\left\{R_0(x^3 - 3L^2x + 2L^3) - P(x - L_1)^3 + PL_2^2(3x - 3L + L_2)\right\}$$

⑤ たわみ角　θ

$$\theta_O = -\frac{L_1L_2^2P}{4EIL}$$

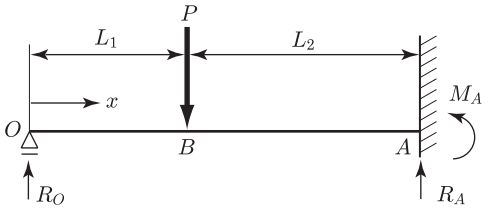
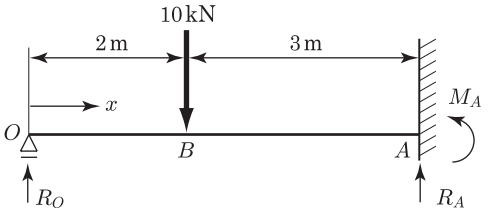


図 1.3.1　荷重図

エクセル例題 1-28-1



$L = 5$ m

$L_1 = 2$ m

$L_2 = 3$ m

$P = 10$ kN

① 反力 R（kN）

$$R_O = \frac{10 \times 9}{2 \times 5^3}(10 + 2) = 4.32$$

$$R_A = 10 - 4.32 = 5.68$$

② せん断力　S（kN）

$$S_{OB} = 4.32$$

$$S_{BA} = 4.32 - 10 = -5.68$$

③ 曲げモーメント　（kN・m）

$$M_A = -\frac{10 \times 2 \times 3}{2 \times 25}(5 + 2) = -8.4$$

$$M_{\max} = \frac{10 \times 2 \times 9}{2 \times 125}(10 + 2) = 8.64 \qquad (x = 2)$$

$$M_{\min} = -\frac{10 \times 2 \times 3}{2 \times 25}(5 + 2) = -8.4 \qquad (x = 5)$$

④ B 点たわみ　δ（m）

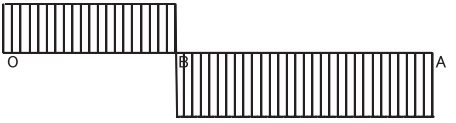
$$\delta_B = \frac{1}{6 \times 9.36 \times 10^4}\left\{4.32(3 \times 5^2 \times 2 - 2^3) - 3 \times 10 \times 3^3 \times 2\right\}$$

$$= 1.78 \times 10^{-6}\,(613.44 - 540) = 1.307 \times 10^{-4}$$

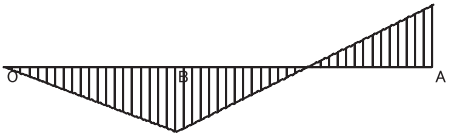
⑤ O 点のたわみ角　θ（rad）

$$\theta_O = -\frac{2 \times 9 \times 10}{4 \times 9.36 \times 10^4 \times 5} = -9.62 \times 10^{-5}$$

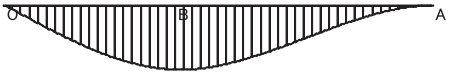
⑥ エクセル出力例



せん断力図 S



曲げモーメント図 M



たわみ図 δ

●軸力 N がある場合

① たわみ　δ

$$\delta_{OB} = \frac{M_A}{N}\left\{\cos k(L-x) - \frac{\sin k(L-x)}{kL} - \frac{x}{L}\right\} + \frac{PL}{N}\left\{\frac{L_1}{L}\frac{\sin k(L-x)}{kL} - \frac{\sin k(L_1-x)}{kL} - \frac{L_2 x}{L^2}\right\} \qquad (0 \leqq x \leqq L_1)$$

$$\delta_{BA} = \frac{M_A}{N}\left\{\cos k(L-x) - \frac{\sin k(L-x)}{kL} - \frac{x}{L}\right\} + \frac{PL_1}{N}\left\{\frac{\sin k(L-x)}{kL} - \frac{L-x}{L}\right\} \qquad (L_1 \leqq x \leqq L)$$

② 曲げモーメント　M

$$M_{OB} = M_A\left\{\cos k(L-x) - \frac{\sin k(L-x)}{kL}\right\} + \frac{P}{k}\left\{\frac{L_1}{L}\sin k(L-x) - \sin k(L_1-x)\right\} \qquad (0 \leqq x \leqq L_1)$$

$$M_{BA} = M_A\left\{\cos k(L-x) - \frac{\sin k(L-x)}{kL}\right\} + PL_1\frac{\sin k(L-x)}{kL} \qquad (L_1 \leqq x \leqq L)$$

ただし

$$M_A = -P\frac{L\sec kL\sin kL_1 - L_1\tan kL}{\tan kL - kL}$$

$$k = \sqrt{\frac{N}{EI}}$$

エクセル例題 1-28-2

$L_1 = 2$ m　　$L_2 = 3$ m　　$L = 5$ m

$N = 5$ kN　　$P = 10$ kN

$k = 7.30882 \times 10^{-3}$ /m　　$kL = 3.654408 \times 10^{-2}$

$$\sin kL = 3.653595 \times 10^{-2} \qquad \cos kL = 9.993323 \times 10^{-1}$$

$$\tan kL = 3.656036 \times 10^{-2} \qquad \sin kL_1 = 1.461711 \times 10^{-2}$$

$$\sec kL = \frac{1}{\cos kL} = 1.0006681$$

$$M_A = -10\frac{5 \times 1.0006681 \times 1.461711 \times 10^{-2} - 2 \times 3.656036 \times 10^{-2}}{3.656036 \times 10^{-2} - 3.654408 \times 10^{-2}}$$

$$= -8.39 \ \mathrm{kN \cdot m}$$

① B 点のたわみ　δ （m）

$$\delta_B = \frac{M_A}{N}\left\{\cos(k \times 3) - \frac{\sin(k \times 3)}{kL} - \frac{2}{L}\right\} + \frac{P \times 2}{N}\left\{\frac{\sin(k \times 3)}{kL} - \frac{3}{N}\right\}$$

$$= \frac{-8.39}{5}\left(9.997596 \times 10^{-1} - 5.999521 \times 10^{-1} - 0.4\right)$$

$$+ 4(5.999521 \times 10^{-1} - 0.6) = 1.31 \times 10^{-4}$$

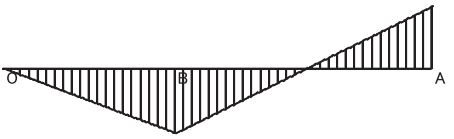
② B 点の曲げモーメント　（kN·m）

$$M_B = M_A\left\{\cos(k \times 3) - \frac{\sin(k \times 3)}{kL}\right\} + P \times 2 \times \frac{\sin(k \times 3)}{kL}$$

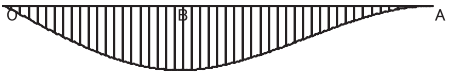
$$= -8.39(9.997596 \times 10^{-1} - 5.999521 \times 10^{-1})$$

$$+ 10 \times 2 \times (5.999521 \times 10^{-1}) = 8.64$$

③ エクセル出力例



曲げモーメント図 M



たわみ図 δ

1.3.2　等分布荷重

① 反力 R

$$R_O = \frac{3w_0L}{8}$$

$$R_A = \frac{5w_0L}{8}$$

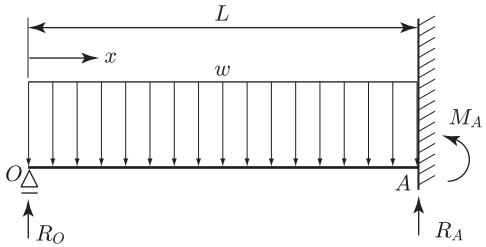


図 1.3.2　荷重図

② せん断力　S

$$S = w_0L\left(\frac{3}{8} - \frac{x}{L}\right)$$

③ 曲げモーメント　M

$$M = \frac{w_0L^2}{8}\left\{3\frac{x}{L} - 4\frac{x^2}{L^2}\right\}$$

$$M_{\max} = \frac{9w_0L^2}{128} \qquad \left(x = \frac{3}{8}L\right), \quad M_{\min} = -\frac{w_0L^2}{8} \qquad (x = L)$$

④ たわみ　δ

$$\delta = \frac{w_0L^4}{48EI}\left\{\frac{x}{L} - 3\left(\frac{x}{L}\right)^3 + 2\left(\frac{x}{L}\right)^4\right\}$$

$$\delta_{\max} = \frac{w_0L^4}{185EI} \qquad (x = 0.4215L)$$

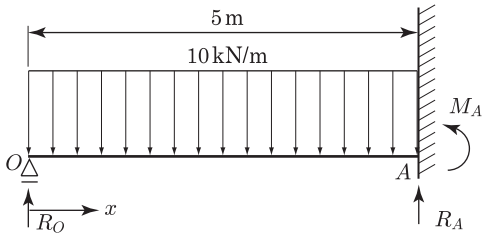
⑤ たわみ角　θ

$$\theta_O = -\frac{w_0L^2}{48EI}$$

エクセル例題 1-29-1

$L = 5$ m

$w = 10$ kN/m



① 反力 R（kN）

$$R_O = \frac{3 \times 10 \times 5}{8} = 18.75, \quad R_A = \frac{5 \times 10 \times 5}{8} = 31.25$$

② せん断力　S（kN）

$$S = 50\left(\frac{3}{8} - \frac{x}{5}\right)$$

③ 最大曲げモーメント　（kN·m）

$$M_{\max} = \frac{9 \times 250}{128} = 17.58 \qquad (x = 1.875)$$

$$M_{\min} = -\frac{250}{8} = -31.25 \qquad (x = 5)$$

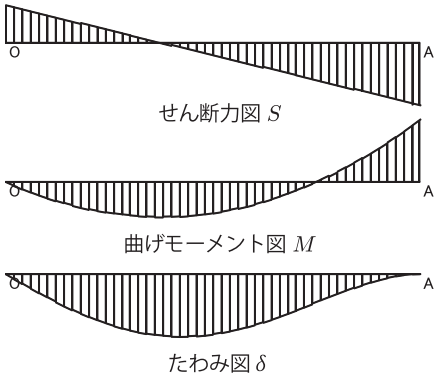
④ 最大たわみ　δ（m）

$$\delta_{\max} = \frac{10 \times 625}{185 \times 9.36 \times 10^4} = 3.61 \times 10^{-4} \qquad (x = 2.11)$$

⑤ O 点のたわみ角　θ（rad）

$$\theta_O = -\frac{250}{48 \times 9.36 \times 10^4} = -5.56 \times 10^{-5}$$

⑥ エクセル出力例



●軸力 N がある場合

① たわみ　δ

$$\delta = \frac{M'}{N}\left\{\cos k(L-x) - \frac{\sin k(L-x)}{kL} - \frac{x}{L}\right\} + \frac{w}{Nk^2}\left\{\cos k(L-x) + \frac{kL}{2}\sin k(L-x) - \frac{1}{2}k^2x(L-x) - 1\right\}$$

② 曲げモーメント　M

$$M = M'\left\{\cos k(L-x) - \frac{\sin k(L-x)}{kL}\right\} + \frac{w}{k^2}\left\{\cos k(L-x) + \frac{kL}{2}\sin k(L-x) - 1\right\}$$

ただし

$$M' = -\frac{wL}{k}\cdot\frac{\tan kL\left(\tan\dfrac{kL}{2} - \dfrac{kL}{2}\right)}{\tan kL - kL}$$

$$k = \sqrt{\frac{N}{EI}}$$

エクセル例題 1-29-2

$L = 5$ m　　$w = 10$ kN/m

$N = 5$ kN

$k = 7.30882 \times 10^{-3}$ /m　　$kL = 3.654408 \times 10^{-2}$

$\sin kL = 3.653595 \times 10^{-2}$　　$\cos kL = 9.993323 \times 10^{-1}$

$\tan kL = 3.656036 \times 10^{-2}$

$$M' = -\frac{10 \times 5}{7.30882 \times 10^{-3}} \times \frac{3.656036 \times 10^{-2}(1.8274074 \times 10^{-2} - 1.827204 \times 10^{-2})}{3.656036 \times 10^{-2} - 3.654408 \times 10^{-2}} = -31.10 \text{ kN·m}$$

① たわみ（$x = 2.5$）　δ（m）

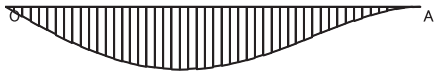
$$\begin{aligned}\delta_{2.5} =& \frac{M'}{N}\left\{\cos(2.5k) - \frac{\sin(2.5k)}{kL} - \frac{2.5}{L}\right)\\ &+ \frac{w}{Nk^2}\left\{\cos(2.5k) + \frac{kL}{2}\sin(2.5k) - \frac{k^2 \times 2.5^2}{2} - 1\right\}\\ =& -6.22(9.998331 \times 10^{-1} - 4.999723 \times 10^{-1} - 0.5)\\ &+ 3.743997 \times 10^4(9.99833 \times 10^{-1}\\ &+ 3.3385 \times 10^{-4} - 1.6693 \times 10^{-4} - 1)\\ =& 4.917 \times 10^{-4}\end{aligned}$$

② 曲げモーメント（$x = 2$）（kN·m）

$$\begin{aligned}M_2 =& M'\left\{\cos 3k - \frac{\sin 3k}{kL}\right\} + \frac{w}{k^2}\left\{\cos 3k + \frac{kL}{2}\sin 3k - 1\right\}\\ =& -31.1\left\{9.997596 \times 10^{-1} - \frac{2.192470 \times 10^{-2}}{3.654408 \times 10^{-2}}\right.\\ &+ \frac{10}{(7.30882 \times 10^{-3})^2}\\ &\left.\left\{9.997596 \times 10^{-1} + \frac{7.30882 \times 10^{-3} \times 5}{2} \times 2.192470 \times 10^{-2} - 1\right\}\right.\\ =& 17.5\end{aligned}$$

③ エクセル出力例

曲げモーメント図 M



たわみ図 δ